Eur. Phys. J. A **3**, 49–63 (1998)

THE EUROPEAN PHYSICAL JOURNAL A
© Springer-Verlag 1998

Renormalization of relativistic self-consistent Hartree-Fock approximation

S.S. Wu, Y.J. Yao

Center for Theoretical Physics and Department of Physics, Jilin University, Changchun 130023, People's Republic of China

Received: 29 August 1997 / Revised version: 30 April 1998
Communicated by P. Schuck

Abstract. The renormalization of the relativistic self-consistent Hartree-Fock approximation is restudied. It is shown that the renormalization procedure suggested by Bielajew and Serot can be greatly simplified and the renormalization achieved in a way no more complicated than that of the relativistic self-consistent Fock approximation, if the parameters in the counterterms are allowed to be density-dependent and the renormalization of the tadpole self-energy is treated appropriately. A transformation relation between the four- and three-dimensional representation of the baryon self-energy is presented and a self-consistent Hartree-Fock scheme different from that considered by Bielajew and Serot studied. The renormalized integral equations for the baryon self-energy which includes effects from the Dirac sea are reformulated in a three-dimensional form. Explicit expressions are derived.

PACS. 21.65.+f Nuclear matter – 21.60.Jz Hartree-Fock and random-phase approximations – 11.10.Gh Renormalization

1 Introduction

Quantum hadrodynamics (QHD) [1] has been applied extensively to the study of nuclear phenomena. At the level of the mean-field and the relativistic Hartree approximation (RHA) it has achieved considerable success in the description of many bulk and single-particle (sp) properties of nuclei. Since it is a renormalizable quantum field theory, in principle it is possible to calculate corrections to the above approximations systematically. However, severe difficulties arise if one intends to take account of quantum corrections which go beyond the one-loop RHA. Owing to the strong couplings between hadronic fields, perturbation theory should not be applied in a naive way. One must at least consider summation of some appropriately selected partial infinite series. But even attempts to formulate the renormalization of the relativistic self-consistent Hartree-Fock approximation (RSHFA) in a way which is convenient for calculations have not yet succeeded. Bielajew and Serot (BS) [2] studied the problem in a comprehensive paper more than a decade ago. As a prototype they considered the relativistic $\sigma-\omega$ model and nuclear matter. Since the ω-field causes no special trouble for the renormalization procedure, for simplicity only the scalar σ-meson exchange was considered as an illustration. RSHFA is shown in Fig.1. In this approximation the baryon propagator is given by

$$G_{HF}(k) = G^0(k) + G^0(k)\Sigma_{HF}(k)G_{HF}(k) \qquad \text{(1a)}$$

$$\Sigma_{HF}(k) = \Sigma^T_{HF}(k) + \Sigma^x_{HF}(k) \qquad \text{(1b)}$$

where the self-energies Σ^T_{HF} and Σ^x_{HF} are contributed by the tadpole and exchange diagram, respectively. The self-consistency considered by BS is realized by using $G_{HF}(k)$ instead of $G^0(k)$ to calculate the tadpole and exchange loops so that $G_{HF}(k)$ occurring in these loops is the same as that given by (1a). One of the chief difficulties in RSHFA is due to the fact that one must simultaneously remove the divergences from all orders in perturbation theory. If Σ^T_{HF} is neglected or need not be considered, the result is referred to as the relativistic self-consistent Fock approximation (RSFA). In this case, the self-energy contributed by the exchange diagram will be denoted simply by Σ_F. Using the spectral representation for the baryon propagator, Wilets and collaborators [3] as well as Bielajew [4] and BS have successfully achieved the renormalization of Σ_F for the case of zero-density. For finite baryon density the formulation of BS is simpler. However, the result of BS for the renormalization of RSHFA is too complicated for practical calculations. The aim of this paper is to find a calculable scheme so that the implication of RSHFA can be studied. In order to avoid unnecessary complications, we shall also consider nuclear matter and the simple model in which baryons couple only with σ-mesons. The Lagrangian density may be written as

$$L_s = -\bar{\psi}(\gamma_\mu\partial_\mu + M)\psi - \frac{1}{2}(\partial_\mu\phi\partial_\mu\phi + m_s^2\phi^2) + g_s\bar{\psi}\psi\phi \qquad \text{(2a)}$$

$$L_{CTC} = \zeta_N\bar{\psi}\gamma_\mu\partial_\mu\psi + M_c\bar{\psi}\psi + \frac{1}{2}\zeta_s\partial_\mu\phi\partial_\mu\phi - \gamma_{sNN}\bar{\psi}\psi\phi + U(\phi) \qquad \text{(2b)}$$

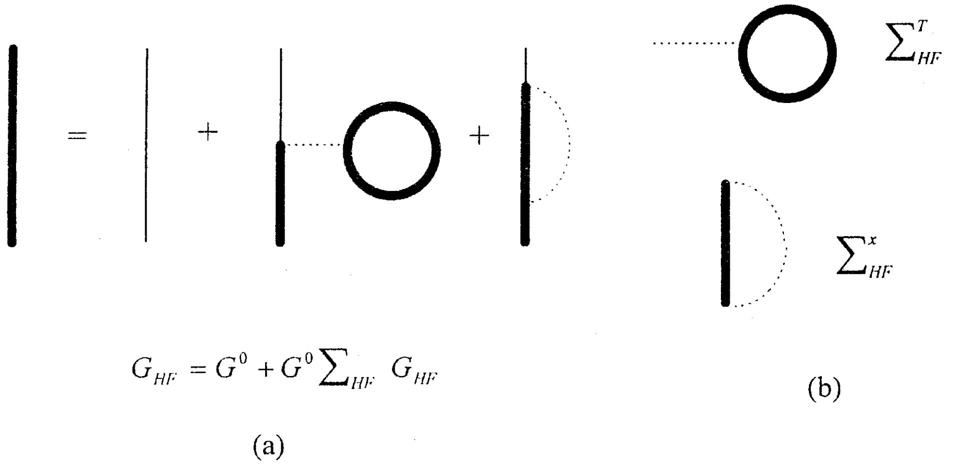

Fig. 1. a The baryon propagator, **b** self-energy in the relativistic self-consistent Hartree-Fock approximation (RSHFA)

$$U(\phi) = \alpha_1\phi + \frac{1}{2!}\alpha_2\phi^2 + \frac{1}{3!}\alpha_3\phi^3 + \frac{1}{4!}\alpha_4\phi^4 \tag{3}$$

where we have used the notation:$\partial_\mu = \partial/\partial_{x_\mu}$,$x_\mu = (\mathbf{x}, ix_0)$ and $x^2 = x_\mu x_\mu = \mathbf{x}^2 - x_0^2$ with $x_0 \equiv t$. In the more stringent sense of multiplicative renormalization, one should include the terms ϕ, ϕ^3 and ϕ^4 in (2a). As it is irrelevant for our present consideration, we shall follow BS and write the Lagrangian density as $L = L_s + L_{CTC}$ where CTC means the counterterm correction. Though $U(\phi)$ itself is physically meaningful and useful [5,6], it is introduced in RHA mainly for the purpose of cancelling the divergences in Σ^T_{HF}. It turns out that a reasonable correction to the energy density is also obtained in this way [7]. Clearly in order to ascertain the meaning of this correction, the additional effect of Σ^x_{HF} should be studied. This has not been done yet. It is known [8,9] that the tadpole self-energy Σ^T caused by the scalar meson can be written rigorously as

$$\Sigma^T = ig_s < \phi(x) >= ig_s < \phi(0) > \tag{4}$$

which is a density-dependent constant. As is shown in [2-4], the renormalization of RSFA can be done neatly and straightforwardly, if the spectral representation technique is used. Why does it become so unthinkably complicated, if in addition $\sum^T_{HF} = ig_s < \phi >_{HF}$ is taken into account, though the latter is but a constant? We would like to demonstrate that the above complication will disappear, if the renormalized parameters in the counterterms are allowed to be density-dependent. As is well known [2], the baryon density may be written as $\rho_B = (-B_\mu B_\mu)^{\frac{1}{2}}$, where in the rest frame of nuclear matter one has $B_\mu = \delta_{\mu 4} i\rho_B$. Thus, no violation of the symmetry properties of L_{CTC} will be caused by such an assumption and the latter may be regarded as an effective way to take account of the density-dependence. In order to simplify the calculation and to take account of the density-dependence in an easier way, there are even suggestions [10] that the parameters in L_s may be density-dependent. Evidently our renormalization procedure applies to this case as well, though for simplicity we shall not consider it here. It will be shown that the renormalization of RSHFA is no more complicated than that of RSFA, if an appropriate treatment of the tadpole renormalization is made. However, the four-dimensional integrals involved in the integral equations are still too complicated to calculate. Thus, a new formulation suiting for practical calculations has to be found. Besides the self-consistent scheme considered by BS in [2], we note that there is the original HF self-consistent scheme, which can be formulated in a form quite similar to the BS consideration. It also achieves a summation of a partial infinite series and thus provides a way to go beyond the simple perturbation calculation. Further we would like to emphasize that it may be regarded as more relevant to the eigenvalue equation, as it refers directly to the self-consistent HF potential and will heareafter be referred to as the potential scheme. It is shown that in this scheme the renormalization of the baryon self-energy becomes much simpler and the integral equaitons obtained are solvable and can be used for practical calculations.

The organization of this paper is as follows. In Sec. 2 the renormalization of RSHFA is considered. Renormalized finite integral equations of the baryon self-energy for both the zero and finite baryon density are derived. The renormalization of the meson propagator in RSHFA is discussed in Sec. 3. In Sec. 4 a transformation relation between the four- and three- dimensional representation of the baryon self-energy is presented and a closed expression for the latter is derived in Sec. 5. In Sec. 6 we shall consider the potential scheme and discuss its relation with the scheme considered by BS. It is emphasized that the relativistic natural orbitals may constitute a more convenient basis for the baryon sp states. Explicit finite three-dimensional integral equations for the renormalized baryon self-energy which includes vacuum polarization effects from the Dirac sea are derived in Sec. 7. In order to cover the case of complex eigenvalues, a biothonormal reperesentation is presented in Sec. 8. The last section contains some concluding remarks and discussion.

2 Renormalization of relativistic self-consistent Hartree-Fock approximation

We shall show that the renormalization procedure for the relativistic self-consistent Hartree-Fock approximation (RSHFA) is almost the same as for the relativistic self-consistent Fock approximation (RSFA) [2-4], if the parameters in the counterterms are allowed to be density-dependent. The baryon propagator is expressed as

$$\begin{aligned} G_{\alpha\beta}(x = x_1 - x_2) &= < T[\psi_\alpha(x_1)\bar{\psi}_\beta(x_2)] > \\ &= \int \frac{d^4k}{(2\pi)^4} e^{ik_\mu x_\mu} G_{\alpha\beta}(k), \end{aligned} \quad (5a)$$

$$\begin{aligned} G(k) &= G^0(k) + G^0(k)\Sigma(k)G(k) \\ &= -\Big[\gamma_\mu k_\mu - iM + \Sigma(k)\Big]^{-1}, \end{aligned} \quad (5b)$$

where we have used the shorthand notation $< O > \equiv < \psi_0|O|\psi_0 >$, $|\psi_0 >$ denotes the exact ground state and (5b) is the Dyson equation. In RSHFA the renormalized baryon self-energy $\Sigma_{HF} = \Sigma^T_{HF} + \Sigma^x_{HF}$ can be written in the form:

$$\begin{aligned} \Sigma^T_{HF} &= -i\frac{g_s^2}{m_s^2} \int \frac{d^\tau q}{(2\pi)^4} e^{iq_0\varepsilon} Tr[G_{HF}(q)] + \Sigma^T_{CTC} \\ &= ig_s < \phi(0) >_{HF} + \Sigma^T_{CTC}, \end{aligned} \quad (6)$$

$$\begin{aligned} \Sigma^x_{HF}(k) &= -g_s^2 \int \frac{d^\tau q}{(2\pi)^4} G_{HF}(q)\Delta^0(k-q) + \Sigma^x_{CTC}(k) \\ &= \hat{\Sigma}^x_{HF}(k) + \Sigma^x_{CTC}(k), \end{aligned} \quad (7)$$

where $\Delta^0(k-q) = -i[(k-q)^2 + m_s^2 - i\varepsilon]^{-1}$ is the non-interacting $\sigma-$meson propagator, $\tau = 4 - \delta$, $\delta \longrightarrow 0+$ and a hat indicates that the quantity has not yet been renormalized. The contribution of the mass counterterm $M_c\bar{\psi}\psi$ to the self-energy can be calculated easily by perturbation theory. It is given by iM_c. Since M_c may be density-dependent, clearly in order to remove the divergence in Σ^T_{HF}, we may simply set $\Sigma^T_{CTC} = iM_T$, where M_T denotes a part of M_c. This shows that there is no need to separate the infinity in Σ^T_{HF} into some partial infinities and then to introduce $U(\phi)$ to cancel them individually. We note that we also need a mass counterterm M_x for the renormalization of Σ^x_{HF}. Since $\Sigma_{HF} = \Sigma^T_{HF} + \Sigma^x_{HF}$, it is obvious that M_T and M_x are additive, i.e. we may set $iM_c = i(M_T + M_x)$, which will not only cancel the infinity in Σ^T_{HF} but also the $k_\mu-$independent divergence in Σ^x_{HF} (see below). From (5b) one has

$$\begin{aligned} G_{HF}(k) = -[&\gamma_\mu k_\mu - iM + ig < \phi >_{HF} \\ &+ iM_T + \Sigma^x_{HF}(k)]^{-1}. \end{aligned} \quad (8)$$

According to the modified Feynman prescription we shall always understand M as $M - i\varepsilon\eta(\mathbf{k}^2, k_0)$, if the latter is not indicated explicitly. Here we have

$$\begin{aligned} \eta(\mathbf{k}^2, k_0) = \frac{1}{2}\{&1 + \text{sign}(|\mathbf{k}| - k_F) \\ &- \text{sign}(k_0)[1 - \text{sign}(|\mathbf{k}| - k_F)]\} \end{aligned} \quad (9)$$

$\varepsilon \longrightarrow 0+$ and sign$x = x/|x|$. Set $M_e = M - g_s < \phi >_{HF} - M_T$. From (8) the Dyson equation may be rewritten in the form

$$G_{HF}(K) = G^0_e(k) + G^0_e(k)\Sigma^x_{HF}(k)G_{HF}(k) \quad (10)$$

$$\begin{aligned} G^0_e(k) &= (\gamma_\mu k_\mu + iM_e)[\frac{-1}{k^2 + M_e^2 - i\varepsilon} \\ &\quad + \frac{\pi i}{E^0_e(k)}\theta(k_F - |\mathbf{k}|)\delta(k_0 - E^0_e(k))] \\ &\equiv G^0_{ev}(k) + G^0_{ed}(k) \end{aligned} \quad (11)$$

where $E^0_e(k) = [\mathbf{k}^2 + M_e^2]^{\frac{1}{2}}$. Since M_e is independent of k_μ, we note that $G^0_{ev}(k) = -[\gamma_\mu k_\mu - i(M_e - i\varepsilon)]^{-1}$ may be interpreted mathematically as a non-interacting propagator of a baryon whose mass has a value of M_e, though M_e is density-dependent. The renormalized M_e is finite and may be regarded as a parameter to be determined by experimental data, for instance, by the saturation properties of nuclear matter or by a variational calculation which minimizes the ground state energy. Let us introduce a new baryon propagator G^v_{HF} defined by

$$G^v_{HF}(k) = G^0_{ev}(k) + G^0_{ev}(k)\Sigma^{xv}_{HF}(k)G^v_{HF}(k) \quad (12)$$

$$\begin{aligned} \Sigma^{xv}_{HF}(k) = -g_s^2 \int &\frac{d^\tau q}{(2\pi)^4} G^v_{HF}(q)\Delta^0(k-q) \\ &+ \Sigma^{xv}_{CTC}(k) \end{aligned} \quad (13)$$

We note that through M_e, G^v_{HF} has already taken the tadpole contribution into account. However, if M_e is understood simply as the mass of a baryon, then from Eqs.(11-13) one sees that G^v_{HF} is equivalent to a baryon propagator at zero-density. It is clear that the renormalization of Σ^{xv}_{HF} can be carried out in the same way as it is for RSFA. Obviously the spectral representation of G^v_{HF} can be written as

$$\begin{aligned} G^v_{HF}(k) &= -Z_2\frac{\gamma_\mu k_\mu + iM_t}{k^2 + M_t^2 - i\varepsilon} \\ &\quad - \int_{m_1^2}^\infty dm^2 \frac{\gamma_\mu k_\mu \alpha_{HF}(-m^2) + iM_t\beta_{HF}(-m^2)}{k^2 + m^2 - i\varepsilon} \end{aligned} \quad (14)$$

where $m_1 = M_t + m_s$ is the threshold of the meson-production continuum. Substituting (14) into (13), one finds that Σ^{xv}_{HF} can be reduced to the following form

$$\begin{aligned} \Sigma^{xv}_{HF}(k) &= \gamma_\mu k_\mu \hat{a}_v(k^2) - iM_e\hat{b}_v(k^2) + \Sigma^{xv}_{CTC}(k) \quad (15a) \\ &= \gamma_\mu k_\mu a_v(k^2) - iM_e b_v(k^2) \quad (15b) \end{aligned}$$

where $\hat{a}_v$ and $\hat{b}_v$ are infinite. For algebraic convenience we shall use the intermediate renormalization [11]. Using the nucleon mass and wavefunction counterterms in L_{CTC}, we may write the counterterm in (15a) as

$$\Sigma^{xv}_{\text{CTC}}(k) = iM_x - \zeta_N\gamma_\mu k_\mu \quad (16)$$

where M_x and ζ_N can be determined by the following renormalization conditions

$$\Sigma^{xv}_{HF}(k)\Big|_{k_\mu} = 0; \frac{\partial}{\partial(\gamma_\nu k_\nu)}\Sigma^{xv}_{HF}\Big|_{k_\mu=0} = 0 \tag{17}$$

From (14–17) and by means of the dimensional regularization as well as Feynman's integral parameterization, one gets

$$\begin{aligned} a_v(k^2) &= \hat{a}_v(k^2) - \hat{a}_v(0) \\ &= -\frac{g_s^2}{16\pi^2}\int_0^\infty dm^2 \int_0^1 dx f_\alpha(-m^2)x \\ &\quad \times \ln\frac{K^2(x,m^2,k^2)}{K^2(x,m^2,0)} \end{aligned} \tag{18a}$$

$$\begin{aligned} b_v(k^2) &= \hat{b}_v(k^2) - \hat{b}_v(0) \\ &= \frac{g_s^2}{16\pi^2}\left[\frac{M_t}{M_e}\right]\int_0^\infty dm^2 \int_0^1 dx f_\beta(-m^2) \\ &\quad \times \ln\frac{K^2(x,m^2,k^2)}{K^2(x,m^2,0)} \end{aligned} \tag{18b}$$

$$\begin{aligned} f_\gamma(-m^2) &= Z_2\delta(m^2 - M_t^2) \\ &\quad +\theta(m^2 - m_1^2)\gamma_{HF}(-m^2) \end{aligned} \tag{18c}$$

$$K^2(x,m^2,k^2) = (1-x)m^2 + xm_s^2 + x(1-x)k^2 \tag{18d}$$

where θ denotes the step function. It is seen that the above results are similar to those obtained in [2] and [4] for RSFA at zero density, except that M_e now contains the contribution from the tadpole self-energy and a different set of renormalization conditions is chosen. Substituting (15b) into (12), we get

$$\begin{aligned} &G^v_{HF}(k) \\ &= -\frac{\gamma_\mu k_\mu(1+a_v(k^2)) + iM_e(1+b_v(k^2))}{k^2(1+a_v(k^2))^2 + M_e^2(1+b_v(k^2))^2 - i\varepsilon} \end{aligned} \tag{19}$$

Hereafter the denominator in (19) will be denoted by $D_v(k^2) - i\varepsilon$, thus $D_v^{(-M_t^2)} = 0$. As pointed out in [4], in order to find additional relations between the two sets (a_v, b_v) and $(\alpha_{HF}, \beta_{HF})$ one may use (14) and the relation $(x - i\varepsilon)^{-1} = P/x + i\pi\delta(x)$. Since α_{HF} and β_{HF} should be real, comparing (19) with (14), we obtain

$$\alpha_{HF}(k^2) = \frac{1}{\pi}\mathrm{Im}\frac{1+a_v(k^2)}{D_v(k^2)} \tag{20a}$$

$$\beta_{HF}(k^2) = \frac{1}{\pi}\left[\frac{M_e}{M_t}\right]\mathrm{Im}\frac{1+b_v(k^2)}{D_v(k^2)} \tag{20b}$$

for $k^2 = \mathbf{k}^2 - k_0^2 < -m_1^2$. (20) shows that $\alpha_{HF} = \beta_{HF} = 0$, if a_v and b_v are real. Since according to (18) a_v and b_v indeed become complex if $k^2 < -m_1^2$, (20) will give us nonzero solutions of α_{HF} and β_{HF}. Either α_{HF} and β_{HF} or a_v and b_v can be solved from (18) and (20) self-consistently [4]. Now let us consider the contribution due to G^0_{ed} [see (11)]. We may define $G^d_{HF}(k)$ by

$$G_{HF}(k) = G^v_{HF}(k) + G^d_{HF}(k) \tag{21}$$

From (7) and (13) we have

$$\Sigma^x_{HF}(k) = \Sigma^{xv}_{HF}(k) + \Sigma^{xd}_{HF}(k), \tag{22}$$

$$\Sigma^{xd}_{HF} = -g_s^2\int\frac{d^4q}{(2\pi)^4}G^d_{HF}(q)\Delta^0(k-q). \tag{23}$$

Following the argument of [2], one can ascertain that $\Sigma^{xd}_{HF}(k)$ is finite. Thus, $\Sigma^x_{CTC} = \Sigma^{xv}_{CTC}$ and one gets (22). It is known [2] that Σ^{xd}_{HF} may be decomposed as

$$\begin{aligned} \Sigma^{xd}_{HF}(k) &= \gamma\cdot\mathbf{k}a_d(\mathbf{k}^2,k_0) + i\gamma_4 k_0 c_d(\mathbf{k}^2,k_0) \\ &\quad -iM_e b_d(\mathbf{k}^2,k_0) \end{aligned} \tag{24}$$

Substituting (22) into (8) and taking account of (9), (15) and (24), we obtain

$$\begin{aligned} G_{HF}(k) &\equiv G^F_{HF}(k) + G^D_{HF} \\ &= (\gamma_\mu k^*_\mu + iM^*_e)[\frac{-1}{k^{*2} + M_e^{*2} - i\varepsilon} \\ &\quad +\theta(k_0)\theta(k_F - |\mathbf{k}|)2\pi i\delta(k^{*2} + M_e^{*2})] \\ &= (\gamma_\mu k^*_\mu + iM^*_e)[\frac{-1}{k^{*2} + M_e^{*2} - i\varepsilon} \\ &\quad +\frac{\pi i}{f'(E_k)E^*_k}\theta(k_F - |\mathbf{k}|)\delta(k_0 - E_k)] \end{aligned} \tag{25}$$

which defines G^F_{HF} and G^D_{HF} and where the starred quantities are given by

$$k^*_\mu = (\mathbf{k}(1+a_v+a_d), ik_0(1+a_v+c_d)) \tag{26a}$$

$$M^*_e = M_e(1+b_v+b_d) \tag{26b}$$

$$E^*_k = E_k(1+a_v+c_d) = \pm[\mathbf{k}^{*2} + M_e^{*2}]^{1/2} \tag{26c}$$

and $f(k_0) = k^*_0 - E^*_k$, $f(E_k) = 0$, $f'(E_k) = \frac{\partial f}{\partial k_0}\Big|_{k_0=E_k}$ with E^*_k taking the positive root. Since $G^d_{HF} = G_{HF} - G^v_{HF}$, using (15), (19), (24) and (25), one finds from (23)

$$\begin{aligned} &a_d(\mathbf{k},k^0) \\ &= -ig_s^2\int\frac{d^4q}{(2\pi)^4}\frac{\mathbf{k}\cdot\mathbf{q}}{\mathbf{k}^2}\cdot\frac{[a_d(\mathbf{q}^2,q_0)D - (1+a_v(q^2))\Delta D]}{D_d D[(k-q)^2 + m_s^2]} \\ &\quad -\frac{g_s^2}{2}\int_0^{k_F}\frac{d^3q}{(2\pi)^3}\frac{\mathbf{k}\cdot\mathbf{q}}{\mathbf{k}^2}\{\frac{1+a_v(q^2)+a_d(\mathbf{q}^2,q_0)}{f'(E_q)E^*_q[(k-q)^2+m_s^2]}\}_{q_0=E_q} \end{aligned} \tag{27a}$$

$$\begin{aligned} &c_d(\mathbf{k}^2,k^0) \\ &= -ig_s^2\int\frac{d^4q}{(2\pi)^4}\frac{q_0}{k_0}\cdot\frac{[c_d(\mathbf{q}^2,q_0)D - (1+a_v(q^2))\Delta D]}{D_d D[(k-q)^2 + m_s^2]} \\ &\quad -\frac{g_s^2}{2}\int_0^{k_F}\frac{d^3q}{(2\pi)^3}\{\frac{1}{k_0 f'(E_q)[(k-q)^2+m_s^2]}\}_{q_0=E_q} \end{aligned} \tag{27b}$$

$$b_d(\mathbf{k},k^0) = ig_s^2\int\frac{d^4q}{(2\pi)^4}\cdot\frac{[b_d(\mathbf{q}^2,q_0)D-(1+b_v(q^2))\Delta D]}{D_dD[(k-q)^2+m_s^2]} + \frac{g_s^2}{2}\int_0^{k_F}\frac{d^3q}{(2\pi)^3}\{\frac{1+b_v(q^2)+b_d(\mathbf{q}^2,q_0)}{f'(E_q)E_q^*[(k-q)^2+m_s^2]}\}_{q_0=E_q} \quad (27c)$$

where we have

$$D\equiv D(q^2)=q^2(1+a_v(q^2))^2+M_e^2(1+b_v(q^2))^2 \quad (28)$$

$$\begin{aligned}D_d &\equiv D_d(\mathbf{q}^2,q_0)\\ &= \mathbf{q}^2(1+a_v(q^2)+a_d(\mathbf{q}^2,q_0))^2\\ &\quad -q_0^2(1+a_v(q^2)+c_d(\mathbf{q}^2,q_0))^2\\ &\quad +M_e^2(1+b_v(q^2)+b_d(\mathbf{q}^2,q_0))^2\\ &\equiv D(q^2)+\Delta D(\mathbf{q}^2,q_0)\end{aligned} \quad (29)$$

Comparing (27) with (3.25) of [2], one notes that in the latter the explicitly k_F-dependent second term in each of (27 a-c) has been missed out through an oversight. Clearly these terms must be considered, otherwise one cannot take the effect of the density distribution into account properly. (27) shows that $\Sigma^{xd}_{HF}(k)$ is indeed finite, as asserted in [2]. It is seen that our renormalization procedure for RSHFA is greatly simplified and its final result is as simple as that of RSFA. Though in principle, (27) can be solved self-consistently by an adequately designed iterating procedure, we note that the four-dimensional integrals in (27) are still too complicated to calculate. In sections starting from four another formulation which is more convenient for calculations will be developed.

According to the theory of renormalization we have made the assertion that the renormalized tadpole self-energy $\Sigma^T_{HF}=ig_s<\phi(0)>_{HF}+iM_T$ may be regarded as a parameter. This is clearly adequate. However, for completeness we would like to remark that in our formalism it is also easy to establish a self-consistent equation for Σ^T_{HF}. Let us replace $i[g_s<\phi(0)>_{HF}+M_T]$ by Σ^T_{HF} in $G_{HF}(k)$ and substitute (21) along with (25) in (6). One obtains

$$\Sigma^T_{HF}=-i\frac{g_s^2}{m_s^2}\int\frac{d^\tau q}{(2\pi)^4}e^{iq_0\varepsilon}Tr\{G^v_{HF}(q) +[G^F_{HF}(q)-G^v_{HF}(q)]+G^D_{HF}(q)\} +\Sigma^T_{CTC} \quad (30a)$$

Evidently the contribution from G^D_{HF} is finite. By means of (18-19) and (26-27) it is not difficult to see that the divergences caused by G^F_{HF} are completely cancelled if Σ^T_{CTC} is chosen as

$$\Sigma^T_{CTC}=i\frac{g_s^2}{M_s^2}\int\frac{d^\tau q}{(2\pi)^4}e^{iq_0\varepsilon}Tr \times\{\sum_{n=0}^{3}\frac{1}{n!}\Big(\frac{\delta^nG^v_{HF}(q)}{\delta(\Sigma^T_{HF})^n}\Big)_{\Sigma^T_{HF}=0}(\Sigma^T_{HF})^n +\sum_{i=1}^{3}(\frac{\delta G^F_{HF}(q)}{\delta a_i})_{a_i=0}\ a_i\} \quad (30b)$$

where for convenience we have denoted a_d, c_d and b_d by $a_i(i=1,2,3)$. With Σ^T_{CTC} given by (30b) we obtain from (30a) a self-consistent equation for Σ^T_{HF}, which is free of divergences and which is similar to (4.25) in [2], though derived in a simple way. However, it is always possible to make another choice of Σ^T_{CTC} which differs from (30b) by a finite constant C. We note that C is well determined, if M_e or $\Sigma^T=i(M-M_e)$ has been chosen empirically. First we would like to point out that according to (4) there should exist a general self-consistent equation for Σ^T. Consider nuclear matter and calculate $<\phi(0)>$, for instance, by the perturbation theory. It is easily seen that $\hat{F}\equiv ig_s<\phi(0)>$ is a function of Σ^T and k_F. If $\hat{F}$ is finite, we immediately obtain $\Sigma^T=\hat{F}(\Sigma^T,k_F)$. However, $\hat{F}$ is divergent. Thus, we have the general renormalized self-consistent equation:

$$\Sigma^T=\hat{F}(\Sigma^T,k_F)+\Sigma^T_{CTC}=F(\Sigma^T,k_F)+C, \quad (31a)$$

where F is the renormalized finite part of $\hat{F}$ and C an undetermined finite constant caused by the indefinite character of ∞. Let $\Sigma^T(k_F^0)$ be determined empirically at the saturation density with $k_F=k_F^0$. According to (31a) C is given by

$$C=\Sigma^T(k_F^0)-F(\Sigma^T(k_F^0),k_F^0), \quad (31b)$$

and one may use (31a) to predict the dependence of Σ^T (or M_e) on k_F. Clearly (31a) reduces to (30a) if RSHFA is considered. Chin [7] derived (31a) in RHA and assumed $C=0$. Horowitz and Serot [12] calculated the HF baryon self-energy under the assumption: $G_{HF}(k)\approx G^D_{HF}(k)$. Their results are thus finite and no Σ^T_{CTC} should be introduced. The density-dependence of the baryon effective mass in nuclear matter has been studied in both [7] and [12]. We would like to investigate the additional effect of $G^F_{HF}(k)$. The value of $M_e(k_F^0)$ at k_F^0 can be determined, for instance, as follows. Consider the Walecka $\sigma-\omega$ model. We may first use the model parameters suggested in [12, 13] to calculate the binding energy per nucleon $E_n=(E/B-M)$ as a function of M_e and k_F. By fitting the experimental value of E_n at k_F^0 we may determine $M_e(k_F^0)$, which can further be used to calculate the bulk symmetry energy a_4 as well as the nuclear compressibility k_V^{-1}. Their values serve to check the adequacy of the value of $M_e(k_F^0)$ and to ask whether a variation of its value may yield a better overall agreement. Certainly, to understand the effect of the variation of parameters better, the model parameters may be readjusted and $M_e(k_F^0)$ refitted. One of our main purposes is to study the effects caused by the renormalized contribution from the Dirac sea.

3 Renormalization of the meson propagator

The renormalization of the meson propagator has also been considered in [2]. In RSHFA the scalar meson propagator is given by

$$\Delta_{HF}(k)=\Delta^0(k)-(2\pi)^4\delta^{(4)}(k)[g_s^{-1}\Sigma^T_{HF}]^2 +\Delta^0(k)\Pi^x_{HF}(k)\Delta^0(k) \quad (32)$$

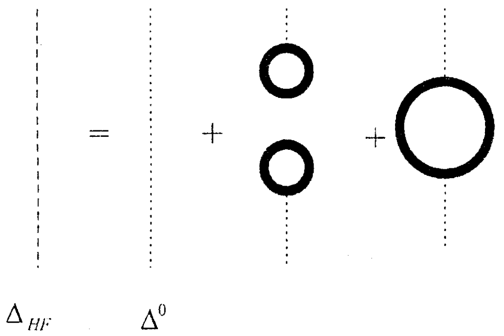

Fig. 2. Diagrammatic representation of the meson propagator in RSHFA, where the heavy solid lines designate G_{HF}

$$\Pi^x_{HF}(k) = +g_s^2 \int \frac{d^\tau q}{(2\pi)^4} Tr[G_{HF}(k+q)G_{HF}(q)] + \Pi^x_{CTC}(k) \tag{33}$$

The diagrammatic representation of $\Delta_{HF}(k)$ is shown in Fig. 2. Since according to (12), (22) and (25) we have

$$G^F_{HF}(k) = G^v_{HF} + G^v_{HF}(k)\Sigma^{xd}_{HF}(k)G^F_{HF}(k) \tag{34}$$

and it has been shown in [2] that $\Sigma^{xd}_{HF}(k)$ behaves as k^{-2} at large k, one sees that the divergence in Π^x_{HF} is sheerly caused by G^v_{HF}. Thus, for the renormalization of Π^x_{HF} one only needs to consider

$$\Pi^{xv}_{HF}(k) = +g_s^2 \int \frac{d^\tau q}{(2\pi)^4} Tr[G^v_{HF}(k+q)G^v_{HF}(q)] + \Pi^x_{CTC}(k) \tag{35}$$

Since $G^v_{HF}(k) \longrightarrow k^{-1}$ for large k, the integral in (35) is quadratically divergent if $\tau \longrightarrow 4$. As we allow that the parameters in L_{CTC} may be density-dependent, there is no need to expand G_{HF} in terms of Σ^T_{HF}. According to (2), Π^x_{CTC} may be written as

$$\Pi^x_{CTC}(k) = i\zeta_s k^2 + i\alpha_2 \tag{36}$$

where ζ_s and $\alpha_2 = \delta m_s^2$ are the meson wavefunction and mass counterterms, respectively. Using the renormalization conditions

$$\Pi^x_{HF}(k)\Big|_{k^2=0} = 0; \frac{\partial \Pi^x_{HF}(k)}{\partial k^2}\Big|_{k^2=0} = 0 \tag{37}$$

ond finds

$$\alpha_2 = -ig_s^2 \int \frac{d^\tau q}{(2\pi)^4} Tr\{[G^v_{HF}(q)]^2\} \tag{38a}$$

$$\zeta_s = -ig_s^2 \Big\{ \frac{d}{dk^2} \int \frac{d^\tau q}{(2\pi)^4} Tr[G^v_{HF}(k+q)G^v_{HF}(q)] \Big\}_{k^2=0} \tag{38b}$$

With Π^x_{CTC} determined by (36) and (38), $\Pi^x_{HF}(k)$ is obviously free of the ultraviolet divergence, i.e. the renormalization is achieved. We note that in order to cancel the divergences in Π^x_{HF} four counterterms are introduced in [2]. In addition to (36), terms associated with α_3 and α_4 in $U(\phi)$ are also needed. It is seen that in our formulation the introduction of these additional terms is unnecessary.

4 A transformation relation

We shall establish a transformation relation between the four- and three-dimensional representation of the baryon self-energy. Though the derivation is elementary, the relation is needed not only to facilitate the calculation but also to ensure that the three-dimensional calculation is worked out in a Lorentz-invariant way. The baryon propagator is given in (5). As is wellknown, $\psi(\mathbf{x}, \mathbf{0})$ may be expanded as

$$\begin{aligned}\psi(\mathbf{x}, 0) &= \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^{3/2}} \Big[b(pr)u(pr)e^{i\mathbf{p}\cdot\mathbf{x}} \\ &\qquad + d^+(pr)v(pr)e^{-i\mathbf{p}\cdot\mathbf{x}} \Big] \\ &\equiv \sum_{r=1}^{4} \int \frac{d^3p}{(2\pi)^{3/2}} c(p_r r)u(p_r r)e^{i\mathbf{p}_r\cdot\mathbf{x}} \\ &= \sum_{r=1}^{4} \int \frac{d^3p}{(2\pi)^{3/2}} c(pr)u(pr)e^{i\mathbf{p}\cdot\mathbf{x}}\end{aligned} \tag{39}$$

where the sp states $\{u(pr)\exp(i\mathbf{p}\cdot\mathbf{x}), r = 1, 2, 3, 4\}$ constitute a complete orthonormal set, $b^+(pr)(d^+(pr))$ denotes a baryon (antibaryon) creation operator, $c(p_r r) = b(pr)$, $\mathbf{p}_r = \mathbf{p}$ if $r = 1, 2$, while $c(p_r r) = d^+(pr-2)$, $u(p_r r) = v(pr-2)$, $\mathbf{p}_r = -\mathbf{p}$ if $r = 3, 4$, because $v(ps)$ $(s = 1, 2)$ denotes a negative energy spinor with momentum $-\mathbf{p}$. The spinors $u(pr)$ satisfy the following completeness and orthonormal conditions

$$\sum_{r=1}^{4} u_\alpha(pr)u^+_\beta(pr) = \delta_{\alpha\beta}, \tag{40a}$$

$$u^+(pr)u(ps) = \delta_{rs}. \tag{40b}$$

Substituting (39) into (5a) and using the translational invariance of the theory, we obtain

$$G_{\alpha\beta}(x_1 - x_2) = \sum_{r,s=1}^{4} \int \frac{d^4p}{(2\pi)^4} u_\alpha(pr)\frac{1}{i}G(pr, ps; p_0)\bar{u}_\beta(ps) \times \exp[ip_\mu(x_1 - x_2)_\mu], \tag{41}$$

which shows that the Fourier transform of $G_{\alpha\beta}(x)$ can be written as

$$G_{\alpha\beta}(p) = \sum_{r,s=1}^{4} u_\alpha(pr)\frac{1}{i}G(pr, ps; p_0)\bar{u}_\beta(ps). \tag{42}$$

In (41-42) we have

$$G(pr, ps; p_0) = i\int_{-\infty}^{\infty} dt e^{ip_0 t} G(pr, ps; t), \tag{43a}$$

$$G(pr, ps; t = t_1 - t_2) = < T[c(pr, t_1)c^+(ps, t_2)] > \tag{43b}$$

and for any operator O we understand $O(t) = \exp(iHt)O\exp(-iHt)$. The Dyson equation for

$G(pr,ps;p_0)$ may be written as

$$G(pr,ps;p_0) = G^0(pr,ps;p_0) - \sum_{\xi,\zeta=1}^{4} G^0(pr,p\xi;p_0)M(p\xi,p\zeta;p_0) \times G(p\zeta,ps;p_0) \quad (44a)$$

$$G^0(pr,ps;p_0) = \delta_{r,s}G^0(pr;p_0) = -\delta_{r,s}[\frac{1-n(pr)}{p_0-E(pr)+i\eta} + \frac{n(pr)}{p_0-E(pr)-i\eta}], \quad (44b)$$

where $n(pr)=1$ if $r=3$ and 4, while for $r=1$ and 2, $n(pr)=0$ or 1 according as (pr) is unoccupied or occupied. Note that for $r=3$ and 4, $E(pr)$ denotes the negative energy. Inserting (44) into (42) and using (5b), we get

$$iG^0_{\alpha\beta}(p) = \sum_{r=1}^{4} u_\alpha(pr)G^0(pr;p_0)\bar{u}_\beta(pr), \quad (45a)$$

$$i\sum_{\xi\zeta} G^0_{\alpha\xi}(p)\Sigma_{\xi\zeta}(p)G^0_{\zeta\beta}(p) = = \sum_{r,s=1}^{4} u_\alpha(pr)G^0(pr;p_0)M(pr,ps;p_0) \times G^0(ps;p_0)\bar{u}_\beta(ps). \quad (45b)$$

By means of (40) we immediately find

$$[i\gamma_4\Sigma(p)]_{\eta\lambda} = \sum_{r,s=1}^{4} u_\eta(pr)M(pr,ps;p_0)u^+_\lambda(ps), \quad (46a)$$

$$M(pr,ps;p_0) = u^+(pr)i\gamma_4\Sigma(p)u(ps). \quad (46b)$$

(46) is the desired relation. The reason why M is referred to as a three-dimensional representation will become clear when we consider its explicit expression. The usefulness of (46) will be demonstrated in the next section. Note that in (46) one should substitute $M(pr,ps;p_0)-u(pr,ps)$ and $\Sigma(p)-\Sigma_u(\mathbf{p})$ for M and Σ, repectively [see, for instance, (53b) and (63) below], if a sp potential u has been introduced to determine the sp states.

5 A closed expression for the self-energy

As illustrated in section 2, in order to proceed with the renormalization procedure we should first find analytic expressions for quantities which are not yet renormalized. For this purpose we only need to consider the Lagrangian density L_s given by (2a) without the counterterms. The corresponding Hamiltonian may be written as

$$H = \sum_{\eta\lambda}(E(\eta)\delta_{\eta\lambda} - u_{\eta\lambda})c^+(\eta)c(\lambda) + \sum_{\mathbf{k}} a^+(\mathbf{k})a(\mathbf{k})\omega(k) - \sum_{\eta\lambda k}\Big[f(\eta\lambda\mathbf{k})c^+(\eta)c(\lambda)a(\mathbf{k}) + \text{h.c.}\Big], \quad (47)$$

where $a^+(\mathbf{k})$ denotes a meson creation operator, $\omega(k) = [\mathbf{k}^2+m_s^2]^{\frac{1}{2}}$, $u_{\eta\lambda}$ an introduced sp potential, for instance the HF potential, $E(\eta)$ the sp energy determined by $u_{\eta\lambda}$ and we have used the shorthand notation: for the baryon indices η, λ, for instance, $\eta=(pr)$, $\sum_\eta = \sum_{r=1}^4 \int d^3p$, while for the meson index $\mathbf{k}$, $\sum_{\mathbf{k}} = \int d^3k$ and

$$f(\eta\lambda\mathbf{k}) = g_s[(2\pi)^3 2\omega(k)]^{-\frac{1}{2}}\bar{u}(\eta)u(\lambda)\delta(\mathbf{q}+\mathbf{k}-\mathbf{p}), \quad (48)$$

It was mentioned in [14] that a closed expression for the self-energy can be derived easily by the method suggested there. Later such an expression has been given in [8] by a different method. Here following [14], we shall derive an expression which is more convenient for our purpose. We have not normal-ordered H, as we shall understand that the vacuum expectation value should be subtracted.

Let us consider the baryon propagator defined in (43b):

$$G(\alpha,\beta;t) = < T[c(\alpha,t_1)c^+(\beta,t_2)] > = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dp_0 e^{-ip_0t}G(\alpha,\beta;p_0). \quad (43b)$$

Since $\frac{i\partial O(t)}{\partial t} = \exp(iHt)[O,H]\exp(-iHt)$ and

$$[c(\alpha),H] = \sum_\eta K_{\alpha\eta}c(\eta) - F(\alpha), \quad (49)$$

$$F(\alpha) = \sum_{\eta\mathbf{k}}\Big[f(\alpha\eta\mathbf{k})c(\eta)a(\mathbf{k}) + f^*(\eta\alpha\mathbf{k})c(\eta)a^+(\mathbf{k})\Big], (50a)$$

$$K_{\alpha\eta} = E(\alpha)\delta_{\alpha\eta} - u_{\alpha\eta}, \quad (50b)$$

we find

$$i\frac{\partial}{\partial t_1}G(\alpha,\beta;t) = i\delta(t)\delta_{\alpha,\beta} + \sum_\eta K_{\alpha\eta}G(\eta,\beta;t) - G(F(\alpha),\beta;t), \quad (51)$$

$$G(F(\alpha),\beta;t) =< T[F(\alpha,t_1)c^+(\beta,t_2)] >, \quad (52)$$

where $F(\alpha,t) = \exp(iHt)F(\alpha)\exp(-iHt)$. The Fourier transform of (51) has the form

$$\sum_\eta(p_0\delta_{\alpha\eta} - K_{\alpha\eta})G(\eta,\beta;p_0) = -\delta_{\alpha,\beta} - G(F(\alpha),\beta;p_0). \quad (53a)$$

In order to make the above equation closed, one may introduce the mass operator M through the following equation

$$G(F(\alpha),\beta;p_0) = -\sum_\eta M(\alpha,\eta;p_0)G(\eta,\beta;p_0). \quad (54)$$

Inserting (54) into (53a), we get

$$G(\alpha,\beta;p_0) = G^0(\alpha,\beta;p_0) - \sum_{\eta\lambda} G^0(\alpha,\eta;p_0) \times \Big[M(\eta,\lambda;p_0) - u_{\eta\lambda}\Big] G(\lambda,\beta;p_0), \quad (53b)$$

which is just (44a). Our aim is to find a closed expression for $M(\eta,\lambda;p_0)$. Now let us operate $i\partial/\partial t_2$ on $G(\alpha,\beta;t)$ and $G(F(\alpha),\beta;t)$, respectively. Taking their Fourier transforms, we obtain

$$\sum_\eta G(\alpha,\eta;p_0)(p_0\delta_{\eta\beta} - K_{\eta\beta}) = -\delta_{\alpha,\beta} - G(\alpha,F(\beta);p_0), \quad (55a)$$

$$\sum_\eta G(F(\alpha),\eta;p_0)(p_0\delta_{\eta\beta} - K_{\eta\beta}) = -g(\alpha,\beta) - G(F(\alpha),F(\beta);p_0), \quad (55b)$$

where $g(\alpha,\beta)$ and the relevant Green functions are as follows:

$$g(\alpha,\beta) = \Big\langle \Big\{F(\alpha), c^+(\beta)\Big\}\Big\rangle = \sum_{\mathbf{k}} \Big\{ f(\alpha\beta\mathbf{k}) < a(\mathbf{k}) > + f^*(\beta\alpha\mathbf{k}) < a^+(\mathbf{k}) > \Big\} \quad (56)$$

$$G(\alpha,F(\beta);t) = \Big\langle T[c(\alpha,t_1)F^+(\beta,t_2)]\Big\rangle, \quad (57a)$$

$$G(F(\alpha),F(\beta);t) = \Big\langle T[F(\alpha,t_1)F^+(\beta,t_2)]\Big\rangle. \quad (57b)$$

Multiplying (54) by $(p_0\delta_{\beta\gamma} - K_{\beta\gamma})$ from the right and using (55), we get

$$\sum_\eta M(\alpha,\eta;p_0)\Big[\delta_{\eta\gamma} + G(\eta,F(\gamma);p_0)\Big] = -g(\alpha,\gamma) - G(F(\alpha),F(\gamma);p_0). \quad (58)$$

Using (54) and $\sum_\xi G(\eta,\xi;p_0)G^{-1}(\xi,\zeta;p_0) = \delta_{\eta\zeta}$,we have

$$\sum_\eta M(\alpha,\eta;p_0)G(\eta,F(\gamma);p_0) = -\sum_{\xi\zeta} G(F(\alpha),\xi;p_0) \times G^{-1}(\xi,\zeta;p_0)G(\zeta,F(\gamma);p_0). \quad (59)$$

From (58) we obtain

$$M(\alpha,\gamma;p_0) = -g(\alpha,\gamma) - G_{ir}(F(\alpha),F(\gamma);p_0), \quad (60a)$$

$$G_{ir}(F(\alpha),F(\gamma);p_0) = G(F(\alpha),F(\gamma);p_0) - \sum_{\xi\zeta} G(F(\alpha),\xi;p_0)G^{-1}(\xi,\zeta;p_0) \times G(\zeta,F(\gamma);p_0). \quad (60b)$$

As shown and emphasized in [14], the complicated second term on the righthand side of (60b) actually need not be calculated, becaused it exactly cancels the reducible diagrams contained in the first term so as to make G_{ir} contain only irreducible diagrams. This is necessary and satisfactory, since otherwise the iterating series derived form (44)or (53b) will contain redundant terms. An irreducible diagram is a connected diagram which cannot be separated into two disjoint parts by cutting any internal line. The above result shows that to calculate G_{ir} only such irreducible diagrams in its first term should be considered. This clearly makes (60) very useful. We note that in the derivation of (60) no approximation has been made. It is not difficult to see that the tadpole contribution is rigorously given by $-g(\alpha,\gamma)$. This is in agreement with what has been found in [8] and [9].

6 Self-consistent HF schemes

From (8) one obtains the following renormalized eigenvalue equation

$$\Big[\gamma_\mu k_\mu - iM_e + \Sigma^x(k)\Big]_{k_0=E_k} u(ks) = 0, \quad (61)$$

where E_k and $u(ks)$ are the eigenvalue and eigenspinor, respectively. (61) is also a pole equation which determines the poles of $G(k)$. The zero-order approximation of $G(k)$ constructed by means of the single-particle (sp) states obtained from (61) will be designated by $G^0_\Sigma(k)$, which satisfies

$$G^0_\Sigma = -\Big[\gamma_\mu k_\mu - iM_e + \Sigma^x(\mathbf{k}.E_k)\Big]^{-1}, \quad (62)$$

$$G(k) = G^0_\Sigma(k) + G^0_\Sigma(k)\Big[\Sigma^x(k) - \Sigma^x(\mathbf{k},E_k)\Big]G(k), \quad (63)$$

where $\Sigma^x(\mathbf{k},E_k)$ represents $\Sigma^x(k)$ at $k_0 = E_k$. The above statement is general. (61) shows that $i\gamma_4\Sigma^x(\mathbf{k},E_k)$ may be interpreted as a generalized HF potential. According to (7) and (10) the self-consistent condition considered by BS may be written as

$$\Sigma^x_{HF}(k) = -g_s^2\int\frac{d^\tau q}{(2\pi)^4} \times\Big\{G^0_e(q) + G^0_e(q)\sum_{n=1}^\infty\Big[\Sigma^x_{HF}(q)G^0_e(q)\Big]^n\Big\} \times\triangle^0(k-q) + \Sigma^x_{CTC}(k) = g_s^2\int\frac{d^\tau q}{(2\pi)^4}\frac{\triangle^0(k-q)}{\gamma_\mu q_\mu - iM_e + \Sigma^x_{HF}(q)} + \Sigma^x_{CTC}(k). \quad (64)$$

(64) is a nonlinear integral equation. Such a type of integral equations has already been studied in the nonrelativistic many-body theory [13]. Their solution contains attractively a whole series of infinite sets of infinitely many diagrams with each set repeating the structure of the previous one. The lowest order approximation $\Sigma^x(k;2)$ to

$\Sigma^x_{HF}(k)$ is obtained by keeping only the first term $G^0_e(q)$ in the curved bracket of (64). Graphically $\Sigma^x(k;2)$ is shown in Fig. 1b by a half-moon like (HML) figure bound by a curved meson line and a baryon line, where the heavy line is replaced by a thin one (see Fig.1a). In contrast, $\Sigma^x_{HF}(k)$ should be represented by a heavy HML figure (see Fig.1b) with the heavy baryon line $G_{HF}(q)$, which, as shown in Fig. 1a, is itself an infinite series of such heavy HML figures. Clearly each of them will again be represented by the above series and this process will continue without an end. Though (64) is quite rich in content, we note that it is only a requirement for the Green function. Besides the general connection that (61) is also the pole equation of the baryon propagator, no additional ties are imposed on them by (64). We would like to point out that there is a self-consistent scheme which is mathematically much simpler and enforces a closer tie between (61) and the HP potential. Let $G^0_\Sigma(k;2)$ be determined by $\Sigma^x(k;2)$ according to (62). Substituting $G^0_\Sigma(q;2)$ for $G_{HF}(q)$ in (7), we obtain a new $\Sigma^x_{HF}(k,\sigma)$, from which we find another $G^0_\sigma(k)$ according to (61) and (62). Let us repeat the above procedure. Replace now $G_{HF}(q)$ by $G^0_\sigma(q)$. From (7) and (62) we get $\Sigma^x_{HF}(k,\sigma^{'})$ and $G^0_{\sigma^{'}}(q)$. They are generally different from the previous set indicated by σ. Thus, there arises the question which one is best to choose. Clearly a reasonable answer is provided by the requirement that one should have $G^0_\sigma(q) = G^0_{\sigma^{'}}(q)$,which is assured by the self-consisent condition

$$\begin{aligned}\Sigma^x_{HF}(k;\sigma) &= -g_s^2\int\frac{d^\tau q}{(2\pi)^4}G^0_\sigma(q)\triangle^0(k-q)+\Sigma^x_{CTC}(k,\sigma)\\ &= g_s^2\int\frac{d^\tau q}{(2\pi)^4}\frac{\triangle^0(k-q)}{\gamma_\mu q_\mu - iM_e+\Sigma^x_{HF}(\mathbf{q},E_q;\sigma)}\\ &\quad+\Sigma^x_{CTC}(k;\sigma)\end{aligned} \tag{65}$$

It is seen that there is only a slight difference between (64) and (65) analytically, though their implications are quite different. On their right-hand sides q_0 in $\Sigma^x_{HF}(q)$ of (64) is a true variable, while there is no q_0 variable in $\Sigma^x_{HF}(\mathbf{q},E_q;\sigma)$ of (65), because it has been set equal to E_q. This makes the analytic structure of the integrand in (65) as a function of q_0 much simpler and the integration over q_0 easier. It will be shown that the transformation relation given in section 4 will provide a convenient way to carry out such an integration. According to (62) we have

$$\begin{aligned}G^0_\sigma(q) &= -\Big[\gamma_\mu q_\mu - iM_e+\Sigma^x_{HF}(\mathbf{q},E_q;\sigma)\Big]^{-1}\\ &= G^0_e(q)+G^0_e(q)\Sigma^x_{HF}(\mathbf{q},E_q;\sigma)G^0_\sigma(q).\end{aligned} \tag{66}$$

Inserting (10) into (9). We obtain

$$\begin{aligned}\Sigma^x_{HF}(k;\sigma) = -g_s^2\int&\frac{d^\tau q}{(2\pi)^4}\\ &\times\Big[G^0_e(q)+G^0_e(q)\Sigma^x_{HF}(\mathbf{q},E_q;\sigma)G^0_\sigma(q)\Big]\\ &\times\triangle^0(k-q)+\Sigma^x_{CTC}(k,\sigma).\end{aligned} \tag{67}$$

Setting $k = (\mathbf{k},E_k)$ in (67), we get a closed equation for $\Sigma^x_{HF}(\mathbf{k},E_k;\sigma)$, which is the HF potential except for a factor $i\gamma_4$. Thus, (65) is really an additional requirement for the determination of the self-consistent HF potential. In the following we shall consider the renormalization of $\Sigma^x_{HF}(k;\sigma)$. Form (66-67) it is seen that the solution to (65) has already achieved a summation of a partial infinite series. Thus, the results derived on the basis of (65), just as of (64), will also go beyond the simple perturbation calculation. Clearly, if $\Sigma(k)$ is independent of k_0, we have $\Sigma(k)-\Sigma(\mathbf{k},E_k)=0$. It is believed in the HF theory that generally $\Sigma_{HF}(\mathbf{k},E_k)$ may be a good approximation to $\Sigma_{HF}(k)$ in (63) [16]. The self-consistent condition (65) suggests that the minimum of $\Delta G_{HF}(k;\sigma^{'}) = G_{HF}(k) - G^0_{\sigma^{'}}(k)$ may be achieved by $\sigma^{'}=\sigma$. However, the above statement is still a conjecture, though it can be tested by numerical calculations. There is some delicacy in the interpretation of $\Sigma(\mathbf{k},E_k)$ in (62). A remark is made in the appendix.

In RSHFA the eigenvalue equation (61) can be written explicitly in the form

$$\begin{aligned}[\gamma\cdot\mathbf{k}A_1(\mathbf{k}^2,k_0)&+i\gamma_4k_0A_2(\mathbf{k}^2,k_0)\\ &-iM_eA_3(\mathbf{k}^2,k_0)]_{k_0=E_k}u(ks)=0,\end{aligned} \tag{68}$$

where according to (26) we have $A_1 = 1+a_v(k^2)+a_d(\mathbf{k}^2,k_0)$, $A_2 = 1+a_v(k^2)+c_d(\mathbf{k}^2,k_0)$ and $A_3 = 1+b_v(k^2)+b_d(\mathbf{k}^2,k_0)$. (18) shows that $A_j (j=1,2,3)$ will become complex for $k^2 < -m_1^2$. Thus the eigenvalue E_k may be complex and the sp basis determined by (68) is no longer convenient for a many-body calculation. Let A^r_j denote the real part of A_j. One may substitute A^r_j for A_j in (68) and the equation obtained in this way will be referred to as the modified HF scheme [16]. Clearly nothing will be changed if A_j is real. One easily finds that the eigenvalue E_k satisfies

$$\begin{aligned}E_k = \pm\Big\{&\Big(\frac{A^r_1(\mathbf{k}^2,k_0)}{A^r_2(\mathbf{k}^2,k_0)}\Big)^2\mathbf{k}^2\\ &+\Big(\frac{A^r_3(\mathbf{k}^2,k_0)}{A^r_2(\mathbf{k}^2,k_0)}\Big)^2M_e^2\Big\}^{\frac{1}{2}}_{k_0=E_k}\end{aligned} \tag{26c}$$

which has already been given in (26c) except for the superscript r. Though for real arguments all the terms on the righthand side of (26c) are real, its root E_k may still be complex. This case will be regarded as abnormal. Here we shall only consider the normal case, i.e. for the A^r_j studied E_k are real and the eigenfunctions $\{u(\eta)=u(ks)\exp(i\mathbf{k}\cdot\mathbf{x})\}$ constitute a complete orthonormal set. Let $c^+(ks)$ denote the creation operator of $u(\eta)$. In this modified scheme the approximate ground state (GS) wavefunction $|\phi_0>$ satisfies

$$\begin{aligned}&c^+(ks)|\phi_0>=0,\quad s=3,4;\\ &c^+(ks)|\phi_0>=0, s=1,2\quad\text{and}\quad|\mathbf{k}|<k_F;\\ &c(ks)|\phi_0>=0, s=1,2\quad\text{and}\quad|\mathbf{k}|>k_F.\end{aligned}$$

Obviously we have

$$<\phi_0|c^+(\mathbf{k}r)c(\mathbf{l}s)|\phi_0>=n(kr)\delta(\mathbf{k}-\mathbf{l})\delta_{rs}. \tag{69}$$

From (43) one has

$$< \psi_0|c^+(\eta)c(\lambda)|\psi_0 >= -\lim_{t\to 0_-} G(\lambda,\eta;t) = \rho_{\eta\lambda}. \quad (70)$$

Since the density matrix $\rho_{\eta\lambda}$ is Hermitian, it can be diagonalized and the set of sp states which diagonalize it is orthonormal. As is wellknown, this set is referred to as the set of natural orbitals (NO) [17], i.e. in the basis of relativistic NO (RNO) we have $\rho_{\eta\lambda} = n(\eta)\delta_{\eta\lambda}$, where $n(\eta)$ is the occupation number of $u(\eta)$. If $|\phi_0 >$ is a good approximation to the exact GS $|\psi_0 >$, (69) shows that the modified HF scheme fulfills the condition of RNO approximately. Thus, it is not merely a simple approximation to RSHFA and may be regarded as a choice made according to RNO, while based on RSHFA. Form (70) it is seen that RNO can also be calculated directly by means of the sp Green function. For a comparison it is worth-while to consider RNO as a choice for the sp basis, especially as it will provide a natural extension to the region of $k^2 < -m_1^2$. Since the Fermi energy $E(k_F)$ is generally much smaller than m_1, it will be real and the expression for G^D_{HF} in (25) remains valid, i.e. our arguments in previous sections are meaningful regardless of the fact that A_j may become complex.

7 Three-dimensional representation

In this section we shall show that in the potential scheme the renormalized integral equations for the baryon self-energy which includes effects from the Dirac sea can be represented in a three-dimensional form. Using (46) and (60), we note that the three-dimensional expression for the not yet renormalized self-energy $\hat{\Sigma}_{HF}(k;\sigma)$ can be obtained in a simple way. Its renormalization is then considered and a self-consistent set of integral equations established. Since according to (8) and (10) the tadpole self-energy has already been taken account of by M_e, we only need to consider $\hat{\Sigma}^x_{HF}(k;\sigma)$. By definition $\hat{\Sigma}^x(k;2)$ and $\hat{\Sigma}^x_{HF}(k;\sigma)$ are obtained by substituting $G^0_e(q)$ and $G^0_\sigma(q)$ for $G_{HF}(q)$ in (7), respectively. From (46), (48) and (60) it is not difficult to see that to find $\hat{\Sigma}^x(k;2)$ and $\hat{\Sigma}^x_{HF}(k;\sigma)$ one only needs to consider the zero-order approximation to $G(F(\alpha),F(\gamma);p_0)$ in (60b). What we have to pay attention to is that for $\hat{\Sigma}^x(k;2)$ we should choose the free-particle states with mass M_e as the sp basis, while for $\hat{\Sigma}^x_{HF}(k;\sigma)$ the sp basis should be the HF states. By means of (50a), (57b) and the following elementary zero-order relations:

$$\Big\langle T\Big[c(prt_1)c^+(qst_2)\Big]\Big\rangle^0 = \delta_{rs}\delta(\mathbf{p}-\mathbf{q})\Big[\theta(t_1-t_2)(1-n(pr)) - \theta(t_2-t_1)n(pr)\Big] \times\exp\Big[-iE(pr)(t_1-t_2)\Big], \quad (71a)$$

where for $r = 1$ and 2 we have $n(pr) = 1$ or 0 according as $|\mathbf{p}| \le k_F$ or $|\mathbf{p}| > k_F$, while $n(pr) = 1$ if $r = 3$ and 4,

$$\Big\langle T\Big[a(\mathbf{k}t_1)a^+(\mathbf{l}t_2)\Big]\Big\rangle^0 = \theta(t_1-t_2)\delta(\mathbf{k}-\mathbf{l})e^{-i\omega(k)(t_1-t_2)}, \quad (71b)$$

$$\Big\langle T\Big[a^+(\mathbf{k}t_1)a(\mathbf{l}t_2)\Big]\Big\rangle^0 = \theta(t_2-t_1)\delta(\mathbf{k}-\mathbf{l})e^{i\omega(k)(t_1-t_2)}, \quad (71c)$$

as well as their Fourier transforms we easily find that (60) gives

$$\begin{aligned}\hat{M}^x(kr,ls;k_0) &= g_s^2\bar{u}(kr)\int\frac{d^3q}{(2\pi)^3}\{\sum_{\lambda=1}^{2}\frac{u(q\lambda)\bar{u}(q\lambda)}{2\omega(k-q)} \\ &\cdot[\frac{1-n(q\lambda)}{k_0-E(q\lambda)-\omega(k-q)+i\eta} + \frac{n(q\lambda)}{k_0-E(q\lambda)+\omega(k-q)-i\eta}] \\ &+\sum_{\lambda=3}^{4}\frac{u(q\lambda)\bar{u}(q\lambda)}{2\omega(k-q)}\cdot\frac{1}{k_0-E(q\lambda)+\omega(k-q)-i\eta}\}u(ks) \\ &\times\delta(\mathbf{k}-\mathbf{l}) \\ &= \hat{M}^x(kr,ks;k_0)\delta(\mathbf{k}-\mathbf{l}).\end{aligned} \quad (72)$$

which shows that $\hat{M}^x$ is of the three-dimensional form. Inserting (72) into (46), we obtain the corresponding expression for $\hat{\Sigma}^x_{HF}(k;\sigma)$ or $\hat{\Sigma}^x(k;2)$ according as $u(kr)$ represents the HF eigenspinor or the free particle spinor. Let us consider $\hat{\Sigma}^x_{HF}(k;\sigma)$, which may be decomposed in the same way as $\Sigma^x_{HF}(k)$, namely

$$\begin{aligned}\hat{\Sigma}^{xv}_{HF}(k;\sigma) &= \hat{\Sigma}^x_{HF}(k;\sigma) - \Sigma^{xd}_{HF}(k;\sigma) \\ &= \gamma_\mu k_\mu \hat{a}_v(k^2) - iM_e\hat{b}_v(k^2),\end{aligned} \quad (73a)$$

$$\begin{aligned}\Sigma^{xv}_{HF}(k;\sigma) &= \hat{\Sigma}^{xv}_{HF}(k;\sigma) + \Sigma^{xv}_{CTC}(k;\sigma) \\ &= \gamma_\mu k_\mu a_v(k^2) - iM_e b_v(k^2),\end{aligned} \quad (73b)$$

$$\begin{aligned}\Sigma^{xd}_{HF}(k;\sigma) &= \gamma\cdot\mathbf{k}a_d(\mathbf{k}^2,k_0) + i\gamma_4 k_0 c_d(\mathbf{k}^2,k_0) \\ &-iM_e b_d(\mathbf{k}^2,k_0),\end{aligned} \quad (73c)$$

where the symbol σ has been suppressed in the factors a, b, and c, since no ambiguity will arise in this section. From (68) and (26) we easily obtain

$$\sum_{\lambda=1}^{2}u(k\lambda)\overline{u}(k\lambda) = (\gamma\cdot\mathbf{k}^* + i\gamma_4E^*_k + iM^*_e(k))(2iE^*_k)^{-1}, \quad (74a)$$

$$\sum_{\lambda=3}^{4}u(k\lambda)\overline{u}(k\lambda) = (-\gamma\cdot\mathbf{k}^* + i\gamma_4E^*_k - iM^*_e(k))(2iE^*_k)^{-1}. \quad (74b)$$

Inserting (74) into (72) and noticing (46b), we get

$$\hat{\Sigma}^x_{HF}(k;\sigma) = -g_s^2 \int \frac{d^3q}{(2\pi)^3}\{\frac{\gamma\cdot\mathbf{q}^* + i\gamma_4 E_q^* + iM_e^*(q)}{4E_q^*\omega(k-q)} \times [\frac{1-n(q)}{k_0 - E_q - \omega(k-q) + i\eta} + \frac{n(q)}{k_0 - E_q + \omega(k-q) - i\eta}] + \frac{\gamma\cdot\mathbf{q}^* + i\gamma_4 E_q^* - iM_e^*(q)}{4E_q^*\omega(k+q)[k_0 + E_q + \omega(k+q) - i\eta]}\} \quad (75)$$

where we have written $n(q\lambda) = n(q)$, $E(q\lambda) = E_q$ for $\lambda = 1$ and 2 and $E(q\lambda) = -E_q$ for $\lambda = 3$ and 4. $\hat{\Sigma}^{xv}_{HF}(k;\sigma)$ is defined and obtained from (75) by setting $n(q) = 0$ and $a_d = c_d = b_d = 0$. Clearly if we delete the asterisks in (75), we get the expression for $\hat{\Sigma}^x(k;2)$. From (73) and (75) one easily finds

$$\hat{a}_v(k^2) = -g_s^2 \int \frac{d^3q}{(2\pi)^3}\frac{k_\mu q_\mu(1+a_v(q^2))}{k^2}[\frac{1}{A} + \frac{1}{B}], \quad (76a)$$

$$\hat{b}_v(k^2) = g_s^2 \int \frac{d^3q}{(2\pi)^3}(1+b_v(q^2))[\frac{1}{A} - \frac{1}{B}], \quad (76b)$$

$$A = 4e_q^*\omega(k-q)[k_0 - e_q - \omega(k-q) + i\eta], \quad (77a)$$

$$B = 4e_q^*\omega(k+q)[k_0 + e_q + \omega(k+q) - i\eta], \quad (77b)$$

where $q_4 = ie_q$ and

$$e_q^* = e_q(1+a_v) = [\mathbf{q}^2(1+a_v)^2 + M_e^2(1+b_v)^2]^{1/2}. \quad (77c)$$

from (76-77) it is seen that both integrals diverge, even though we have assumed that a_v and b_v inside these integrals are renormalized and finite. According to (73a) the explicit expression of $\hat{\Sigma}^{xd}_{HF}(k;\sigma)$ can be found straightforwardly from (75) and (76). It will be shown that it is finite as asserted in [2].

For the renormalization of $\hat{\Sigma}^{xv}_{HF}(k;\sigma)$ we shall follow the procedure described in Sec. 2. The counterterm can be written as

$$\Sigma^{xv}_{CTC}(k;\sigma) = iM_x - \zeta_N\gamma_\mu k_\mu \quad (78)$$

where the parameters M_x and ζ_N are determined by the intermediate renormalization conditions. Making use of the latter and (76), we have

$$a_v(k^2) = \hat{a}_v(k^2) - \hat{a}_v(0) = -g_s^2 \int \frac{d^3q}{(2\pi)^3} \cdot \frac{k^2}{2\omega(q)(e(q)+\omega(q))^2[k^2 + (e(q)+\omega(q))^2 - i\eta]}, \quad (79a)$$

$$b_v(k^2) = \hat{b}_v(k^2) - \hat{b}_v(0) = g_s^2 \int \frac{d^3q}{(2\pi)^3} \cdot \frac{1+b_v(q^2)}{2e^*(q)\omega(q)(e(q)+\omega(q))} \cdot \frac{k^2}{k^2 + (e(q)+\omega(q))^2 - i\eta}. \quad (79b)$$

In the derivation of (79) we have made use of the fact that a_v and b_v depend only on $k^2 = \mathbf{k}^2 - k_0^2$. Thus we may first set $\mathbf{k}^2 = 0$. After we have got the final result, which is now a function of k_0^2, we then change k_0^2 to $-k^2 = k_0^2 - \mathbf{k}^2$. It is not difficult to see that (79) now gives a closed set of finite integral equations for the renormalized a_v and b_v. Our main purpose is to find a calculable scheme for the density-dependent case. If $\sum^{xd}_{HF}(k;\sigma)$ is finite, the counterterms for $\hat{\Sigma}^x_{HF}(k;\sigma)$ and $\hat{\Sigma}^{xv}_{HF}(k;\sigma)$ should be the same. This can be checked by means of (75). Indeed, we have

$$\Sigma^{xd}_{HF}(k;\sigma) = \Sigma^x_{HF}(k;\sigma) - \Sigma^{xv}_{HF}(k;\sigma) = \hat{\Sigma}^x_{HF}(k;\sigma) - \hat{\Sigma}^{xv}_{HF}(k;\sigma). \quad (80)$$

From (75) we easily find

$$\mathbf{k}^2 a_d(\mathbf{k}^2, k_0) = -g_s^2 \int \frac{d^3q}{(2\pi)^3} \times\{(\frac{\mathbf{k}\cdot\mathbf{q}}{4E^*(q)}[1 + a_v(q^2) + a_d(\mathbf{q}^2, q_0)] \times[P(k,q,n(q)) + Q(k,q)])_{q_0=E(q)} - (\frac{\mathbf{k}\cdot\mathbf{q}[1+a_v(q^2)]}{4e^*(q)}f_+(k,q))_{q_0=e(q)}\}, \quad (81a)$$

$$k_0 c_d(\mathbf{k}^2, k_0) = -g_s^2 \int \frac{d^3q}{(2\pi)^3} \times\{\frac{1}{4}[P(k,q,n(q)) + Q(k,q)]_{q_0=E(q)} - \frac{1}{4}f_+(k,q)_{q_0=e(q)}\}, \quad (81b)$$

$$b_d(\mathbf{k}^2, k_0) = g_s^2 \int \frac{d^3q}{(2\pi)^3}\{(\frac{1+b_v(q^2)+b_d(\mathbf{q}^2,q_0)}{4E^*(q)} \times[P(k,q,n(q)) - Q(k,q)])_{q_0=E(q)} - (\frac{1+b_v(q^2)}{4e^*(q)}f_-(k,q))_{q_0=e(q)}\}, \quad (81c)$$

where we have

$$P(k,q,n(q)) = \frac{1}{\omega(k-q)}\left[\frac{1-n(q)}{k_0 - E(q) - \omega(k-q) + i\eta} + \frac{n(q)}{k_0 - E(q) + \omega(k-q) - i\eta}\right], \quad (82a)$$

$$Q(k,q) = \{\omega(k+q)[k_0 + E(q) + \omega(k+q) - i\eta]\}^{-1}, \quad (82b)$$

$$f_\pm(k,q) = \{\omega(k-q)[k_0 - e(q) - \omega(k-q) + i\eta]\}^{-1} \pm\{\omega(k+q)[k_0 + e(q) + \omega(k+q) - i\eta]\}^{-1}. \quad (83)$$

(81) is the set of integral equaitons for the determination of $\Sigma^{xd}_{HF}(k;\sigma)$. Let a_d, c_d and b_d be denoted by $a_i (i = 1,2,3)$. If the integrals on the righthand side of

(81) exist, it is seen that $\lim_{|k|\to\infty} a_i(\mathbf{k}^2, k_0 = E(k)) \longrightarrow 0$ with a quite fast speed. On the other hand, this property of a_i ensures that the two terms in the integrand of each integral in (81) cancel each other for large $|\mathbf{q}|$. This means that all the integrals in (81) are convergent. (81) may be solved by an iterative procedure. The integrals involved are clearly calculable. For instance, for a spherically symmetric problem every three-dimensional integral in (81) can be reduced without much difficulty to an integral of one dimension.

After (81) is established, we note that there is no necessity to calculate a_v and b_v by means of (79). In the appendix we have shown that $G^0_\sigma(k)$ can be written as

$$G^0_\sigma = \frac{-1}{\gamma_\mu k_\mu - i\tilde{M}_e} = -\frac{\gamma_\mu k_\mu + i\tilde{M}_e}{k^2 + \tilde{M}_e^2 - i\varepsilon}, \tag{84a}$$

$$\tilde{M}_e = M_e(1 + b_v(\tilde{k}^2))/(1 + a_v(\tilde{k}^2)), \tag{84b}$$

where $\tilde{k}^2 = \mathbf{k}^2 - E_k^2$. It has been pointed out in the appendix that $\tilde{k}^2$ is actually a constant independent of $\mathbf{k}^2$, because it is a root of the following equation

$$\tilde{k}^2(1 + a_v(\tilde{k}^2))^2 + M_e^2(1 + b_v(\tilde{k}^2))^2 = 0. \tag{85}$$

Thus, the HF potential only effects a change of the mass and $G^0_\sigma(k)$ may be regarded as a free baryon propagator with an effective mass $\tilde{M}_e$. Substituting $G^0_\sigma(k)$ into (65) and using the Feynman integral parameterization, we get

$$\Sigma^{xv}_{HF}(k;\sigma) = -ig_s^2 \int_0^1 dx \int \frac{d^\tau Q}{(2\pi)^4} \cdot \frac{x\gamma_\mu k_\mu + i\tilde{M}_e}{[Q^2 + K^2(x,k^2)]^2} + \Sigma^{xv}_{CTC}((k;\sigma), \tag{86a}$$

$$K^2(x,k^2) = x(1-x)k^2 + (1-x)\tilde{M}_e^2 + xm_s^2. \tag{86b}$$

By means of the dimensional regularization and the intermediate renormalization we obtain

$$a_v(k^2) = \hat{a}_v(k^2) - \hat{a}_v(0) = \frac{g_s^2}{16\pi^2}\int_0^1 dx x \ln\frac{K^2(x,0)}{K^2(x,k^2)}, \tag{87a}$$

$$b_v(k^2) = \hat{b}_v(k^2) - \hat{b}_v(0) = \frac{g_s^2}{16\pi^2}\int_0^1 dx \frac{1 + b_v(\tilde{k}^2)}{1 + a_v(\tilde{k}^2)} \ln\frac{K^2(x,k^2)}{K^2(x,0)}. \tag{87b}$$

It is seen that (85) and (87) build a closed set of equations. Further we note that the integration over x in (87) can be carried out analytically. Thus, (87) acturally involves no integration and is much easier to handle, especially if one wants to consider the case of complex $a_\sigma, b_\sigma, (\sigma = v$ and $d), c_d$ and $E(kr)$. Obviously, (79) and (87) may also serve as a cross check.

8 Biorthonormal representation

If E_k is complex, the sp states with different momenta $\mathbf{k}$ are still orthogonal to each other in the case of nuclear matter, but (40) no longer holds between positive and negative energy states. Therefore, in order to cover this case, our above formulation must be generalized. Here we shall consider the biorthonormal representation. Let $\{w(pr)\}$ be defined by

$$w^+(ps)u(pr) = u^+(pr)w(ps) = \delta_{rs}, \tag{88a}$$

i.e. $\{w(ps)\}$ is the set biorthonormal to $\{u(pr)\}$. Obviously $w(pr) = u(pr)$ if $\{u(pr)\}$ is already an orthonormal set. The creation operator $c^+(pr)[f^+(qs)]$ of the sp state $u(pr)\exp(i\mathbf{p}\cdot\mathbf{x})[w(qs)\exp(i\mathbf{q}\cdot\mathbf{x})]$ satisfies the anticommutation relations

$$\begin{aligned} &\{f^+(qs), c(pr)\} = \{f(qs), c^+(pr)\} = \delta_{sr}\delta^3(\mathbf{q}-\mathbf{p}), \\ &\{f^+(qs), f^+(pr)\} = \{c^+(qs), c^+(pr)\} = 0. \end{aligned} \tag{89}$$

The relation for the annihilation operators is obtained through the hermitian conjugation. The completeness relation reads

$$\sum_{r=1}^4 u_\alpha(pr)w^+_\beta(pr) = \delta_{\alpha\beta}. \tag{88b}$$

Besides(39), the Dirac field operator can also be expanded as

$$\psi(\mathbf{x},0) = \sum_{r=1}^4 \int \frac{d^3p}{(2\pi)^{3/2}} f(pr)w(pr)e^{i\mathbf{p}\cdot\mathbf{x}}, \tag{90}$$

and the Hamiltonian may be written in the form

$$H = \sum_{\mathbf{k}} a^+(\mathbf{k})a(\mathbf{k})\omega(k) + \sum_{\eta\lambda}(E(\eta)\delta_{\eta\lambda} - u_{\eta\lambda})f^+(\eta)c(\lambda) - \sum_{\eta\lambda\mathbf{k}} h(\eta\lambda\mathbf{k})f^+(\eta)c(\lambda)[a(\mathbf{k}) + a^+(-\mathbf{k})], \tag{91a}$$

$$h(\eta\lambda\mathbf{k}) = g_s[(2\pi)^3 2\omega(k)]^{-\frac{1}{2}}\bar{w}(pr)u(qs) \times\delta(\mathbf{q}+\mathbf{k}-\mathbf{p}). \tag{91b}$$

(89) shows that the creation operator corresponding to $c(pr)[f(pr)]$ is $f^+(pr)[c^+(pr)]$. The zero-order approximation to the ground state wavefunction now satisfies

$$\begin{aligned} &f^+(pr)|\phi_0> = 0 \qquad r = 3,4,\ \text{all}\ \ \mathbf{p}, \\ &f^+(pr)|\phi_0> = 0 \qquad r = 1,2,\ |\mathbf{p}| < k_F, \\ &c(pr)|\phi_0> = 0 \qquad r = 1,2,\ |\mathbf{p}| > k_F. \end{aligned} \tag{92}$$

Substituting (90) into (5a), we obtain

$$\begin{aligned} G_{\alpha\beta}(x = x_1 - x_2) &= < T[\psi_\alpha(x_1)\bar{\psi}_\beta(x_2)] > \\ &= \int \frac{d^4p}{(2\pi)^4} e^{ip_\mu x_\mu} \sum_{r,s=1}^4 u_\alpha(pr)\frac{1}{i}G(pr,ps;p_0)\bar{w}(ps), \end{aligned}$$

which implies

$$G_{\alpha\beta}(p) = \sum_{r,s=1}^{4} u_\alpha(pr)\frac{1}{i}G(pr,ps;p_0)\bar{w}_\beta(ps), \quad (93a)$$

where we have

$$G(pr,ps;p_0) = i\int_{-\infty}^{\infty} dte^{ip_0t} \times < T[c(prt_1)f^+(pst_2)] > . \quad (93b)$$

The baryon number operator has the form

$$B = \int d^3x\psi^+(\mathbf{x},0)\psi(\mathbf{x},0) = \sum_\eta f^+(\eta)c(\eta), \quad (94)$$

which clearly commutes with H [see (91)] and is a constant of motion. The contraction $\underbrace{c(prt_1)f^+(qst_2)}$ is given by the zero-order approximation to $G(pr,qs;t) = < T[c(prt_1)f^+(qst_2)] >$. By means of (92) one easily finds

$$G^0(pr,qs;t) = \delta_{rs}\delta(\mathbf{p}-\mathbf{q})G^0(pr;t), \quad (95a)$$

$$G^0(pr;t) = [\theta(t_1-t_2)(1-n(pr)) -\theta(t_1-t_2)n(pr)]e^{-iE(pr)t}. \quad (95b)$$

Obviously the contractions $\underbrace{cc}$ and $\underbrace{f^+f^+}$ are zero. We note that the Fourier transform of (95) can still be written in the form of (44b) even if $E(pr) = E_{re}(pr) + iE_{im}(pr)$ with $E_{im}(pr) \neq 0$. In this case we may set $\eta = 0$. We have to distinguish two cases: (1) $E_{im}(pr) > 0$ and (2) $E_{im}(pr) < 0$. According to [18] and from (95), we obtain the results:
(1) $E_{im}(pr) > 0$. In this case we have

$$G^0(pr;t) = \frac{1}{2\pi i}\left\{\int_{-\infty+iR}^{\infty+iR} dp_0\frac{-(1-n(pr))}{p_0-E(pr)}e^{-ip_0t} + \int_{-\infty}^{\infty} dp_0\frac{-n(pr)}{p_0-E(pr)}e^{-ip_0t}\right\}, \quad (96a)$$

where R is a real number and satisfies $R > E_{im}(pr)$.
(2) $E_{im}(pr) < 0$. Here one obtains

$$G^0(pr;t) = \frac{1}{2\pi i}\left\{\int_{-\infty}^{\infty} dp_0\frac{-(1-n(pr))}{p_0-E(pr)}e^{-ip_0t} + \int_{-\infty-iR}^{\infty-iR} dp_0\frac{-n(pr)}{p_0-E(pr)}e^{-ip_0t}\right\}, \quad (96b)$$

where one should require $R + E_{im}(pr) > 0$. (96) shows that (44b) is also valid for complex $E(pr)$, if proper care is taken over the contour integration. The occupation number $n(pr)$ takes the same value as in (44b), though it is now the zero-order approximation to $< f^+(pr)c(pr) >$. The Dyson equation for $G(pr,ps;p_0)$ may also be written in the form of (44a). From (93a) one again obtains (45) adn (46) except that now one should substitute $\bar{w}$ for $\bar{u}$ in (45), w^+ for u^+ in (46). From (89) and (91) we have

$$[c(\alpha),H] = \sum_\eta(E(\alpha)\delta_{\alpha\eta} - u_{\alpha\eta})c(\eta) - X(\alpha), \quad (97a)$$

$$X(\alpha) = \sum_{\eta\mathbf{k}} h(\alpha\eta\mathbf{k})c(\eta)[a(\mathbf{k}) + a^+(-\mathbf{k})]; \quad (97b)$$

$$[f^+(\beta),H] = -\sum_\lambda(E(\beta)\delta_{\lambda\beta} - u_{\lambda\beta})f^+(\lambda) + W(\beta), \quad (98a)$$

$$W(\beta) = \sum_{\lambda\mathbf{l}} h(\lambda\beta\mathbf{l})f^+(\lambda)[a(\mathbf{l}) + \mathbf{a}^+(-\mathbf{l})]. \quad (98b)$$

Following the same procedure as described in section 5, we obtain

$$M(\alpha,\gamma;p_0) = -g(\alpha,\gamma) - G_{ir}(X(\alpha),W(\gamma);p_0), \quad (99a)$$

$$G_{ir}(X(\alpha),W(\gamma);p_0) = G(X(\alpha),W(\gamma);p_0) - \sum_{\xi\zeta} G(X(\alpha),\xi;p_0)G^{-1}(\xi\zeta;p_0) \times G(\zeta,W(\gamma);p_0). \quad (99b)$$

It is seen that (99) has the same form as (60). Indeed we have $g(\alpha,\gamma) =< \{X(\alpha),f^+(\gamma)\} >$, while the expressions for the other terms can be obtained from (60) by substituting $X(\alpha)$ for $F(\alpha)$ and $W(\gamma)$ for $F^+(\gamma)$. Following (96), we shall understand that proper attention has been paid to the integration contour. From (99) one immediately finds that the expression for $M^x(kr,ks;k_0)$ can be obtained from (72) simply by substituting $\bar{w}$ for $\bar{u}$, even if the eigenvalue becomes complex. This implies that (75) is also valid for complex $E(pr)$, if we note that in this case the correct expression for (74) should be the one where $\bar{u}$ is replaced by $\bar{w}$. Since (79) and (81) follow straightforwardly from (75), one concludes that they remain valid even if $a_\sigma, b_\sigma(\sigma = v$ and $d)$, c_d and $E(pr)$ are complex.

9 Concluding remarks and discussion

We have demonstrated and emphasized the appropriateness and advantage of the assumption that the parameters in the counterterms may be density-dependent. Under this assumption we have shown that the renormalization of RSHFA can be worked out in a way no more complicated than that of RSFA. It is known that the baryon propagator is an important elementary building brick for the relativistic many-body calcualtion. In order to be able to calculate the correlation and medium effects one would like to find for it an expression which is not only a good approximation but also easy to handle. We have emphasized that the self-consistent scheme considered by BS is different from the original HF scheme, which is much simpler and can be formulated in a way quite similar to the BS consideration. It has been referred to as the potential scheme in order to make a distinction between different HF schemes. In Sec. 6 we have suggested that the minimum of $\Delta G_{HF}(k,\sigma') = G_{HF}(k) - G^0_{\sigma'}(k)$ may be attained through the self-consistent condition (65). If $G^0_\sigma(k)$ is a good approximation to $G_{HF}(k)$, then we may substitute $G^0_\sigma(k)$ for $G_{HF}(k)$. Since $G^0_{HF}(k)$ is much simpler than $G_{HF}(k)$ [see (63)], this shows the advantage of the potential scheme. Indeed, if $G^0_\sigma(k) \approx G_{HF}(k)$, we have (65) $\approx$ (7) and the self-consistent condition studied by BS is also solved. In addition, (67) demonstrates that the solution to (65) has already achieved a summation of a partial infinite series. Thus, it is important to study the potential scheme first. From the three-dimensional representation (81) it is seen that this scheme provides not only a calculable renormalization procedure which takes account of vacuum polarization effects from the Dirac sea properly but also a way to go beyond the simple perturbation calculation. A numerical solution to (81) will be presented in a succeeding paper.

We are very grateful to Profs. L. M. Yang, Y. B. Dai, R. H. Wang and J. C. Su for valuable discussions. This work is supported in part by the National Natural Science Foundation of China and the Foundation of State Education Commission.

A remark on $\Sigma(\mathbf{k}, E_k)$ in $G^0_\Sigma(k)$

A remark should be made on how to understand $\Sigma(\mathbf{k}, E_k)$ in (62). For this purpose we only need to consider the case of zero-density. We shall use $G^0_\sigma(k)$ to denote a typical $G^0_\Sigma(k)$ obtained in the iterating procedure described in section 6. Let us start with $\Sigma^{xv}(k;2)$ as the initial input. By definition $\Sigma^{xv}(k;2)$ is given by

$$\Sigma^{xv}(k;2) = g_s^2 \int \frac{d^\tau q}{(2\pi)^4} G^0_e(q) \frac{i}{(k-q)^2 + m_s^2 - i\varepsilon} + \Sigma^{xv}_{CTC}(k;2), \quad \text{(A1a)}$$

$$G^0_e(q) = \frac{-1}{\gamma_\mu k_\mu - iM_e} = -\frac{\gamma_\mu k_\mu + iM_e}{k^2 + M_e^2 - i\varepsilon}. \quad \text{(A1b)}$$

(A1) shows that $\Sigma^{xv}(k;2)$ can be written as

$$\Sigma^{xv}(k;2) = \gamma_\mu k_\mu a_0(k^2) - iM_e b_0(k^2). \quad \text{(A2)}$$

As is wellknown , the eigenvalue of (61) with $\Sigma(k) = \Sigma^{xv}(k;2)$ is determined by

$$E_k(2) = \pm[\mathbf{k}^2 + M_e^2([1+b_0]/[1+a_0])^2]^{1/2}, \quad \text{(A3)}$$

where the argument of either a_0 or b_0 is $\tilde{k}^2 = \mathbf{k}^2 - E_k(2)^2$ [see (26)]. Substituting (A2) and the positive $E_k(2)$ into (62), we get

$$G^0_\Sigma(k;2) = \frac{\gamma\cdot\mathbf{k}(1+a_0) + i\gamma_4(k_0 + a_0E_k(2)) + iM_e(1+b_0)}{(k_0 + a_0E_k(2))^2 - (1+a_0)^2E_k(2)^2}. \quad \text{(A4)}$$

We observe that $E_k(2)$ is indeed a pole of $G^0_\Sigma(k;2)$, but the other pole is $-(1+2a_0)E_k(2)$ rather than $k_0 = -E_k(2)$. This is certainly not desirable. Although the expression for $G^0_\Sigma(k)$ is correct, we note that the simple-minded way of evaluating $\Sigma(\mathbf{k}, E_k)$ is misleading. By definition $G^0_\Sigma(k;2)$ is the zero-order approximation to $G(k;2) = -[\gamma_\mu k_\mu - iM_e + \Sigma^{xv}(k;2)]^{-1}$ and is constructed by the sp states obtained from (61). By means of (44–45), it can also be calculated as follows:

$$\begin{aligned}
G^0_\Sigma(k;2) &= i\sum_{r=1}^{4} u(kr)\bar{u}(kr)[\frac{1-n(kr)}{k_0 - E(kr) + i\varepsilon} + \frac{n(kr)}{k_0 - E(kr) - i\varepsilon}] \\
&= i\frac{\gamma\cdot\mathbf{k}^* + i\gamma_4E^*_k(2) + iM^*_e}{2iE^*_k(2)(k_0 - E_k(2) + i\varepsilon)} + i\frac{-\gamma\cdot\mathbf{k}^* + i\gamma_4E^*_k(2) - iM^*_e}{2iE^*_k(2)(k_0 + E_k(2) - i\varepsilon)} \\
&= \frac{\gamma_\mu k_\mu + iM_e([1+b_0]/[1+a_0])}{k_0^2 - (E_k(2) - i\varepsilon)^2} \\
&= \frac{-1}{\gamma_\mu k_\mu - iM_e([1+b_0]/[1+a_0])},
\end{aligned} \quad \text{(A5)}$$

where $E^*_k(2) = (1+a_0)E_k(2)$ and $E(kr) = E_k(2)$ if $r=1$ and 2, while $E(kr) = -E_k(2)$ if $r=3$ and 4. (A5) is the desired result. It takes account of the correct pole property from the beginning. According to (A3), we note that $\tilde{k}^2$ should be a root of the following equation

$$\tilde{k}^2(1 + a_0(\tilde{k}^2))^2 + M_e^2(1 + b_0(\tilde{k}^2))^2 = 0. \quad \text{(A6)}$$

Since a_0 and b_0 depend only on $\tilde{k}^2$, (A6) shows that $\tilde{k}^2$ is a constant independent of $\mathbf{k}^2$. Comparing (A5) with (A1b), one sees that $G^0_\Sigma(k;2)$ may be regarded as a free baryon propagator with an effective mass $M_e([1+b_0]/[1+a_0])$. Thus, if we now substitue $G^0_\Sigma(k;2)$ for $G_{HF}(q)$ in (7) and start a new round of iteration, we obtain (A2–A6) again, though with new a and b replacing a_0 and b_0, respectively. By means of the method of deduction from n to $n+1$, one concludes that generally one has

$$G^0_\sigma(k) = -\{\gamma_\mu k_\mu - iM_e \times([1 + b_v(\tilde{k}^2)]/[1 + a_v(\tilde{k}^2)])\}^{-1}, \quad \text{(A7a)}$$

$$\Sigma^{xv}_{HF}(k;\sigma) = \gamma_\mu k_\mu a_v(k^2) - iM_e b_v(k^2), \quad \text{(A7b)}$$

$$E_k = \pm\{\mathbf{k}^2 + M_e^2([1 + b_v(\tilde{k}^2)]/[1 + a_v(\tilde{k}^2)])^2\}^{1/2}, \quad \text{(A7c)}$$

where $\tilde{k}^2 = \mathbf{k}^2 - E_k^2$ and following (9), we have added the subscript HF to $\Sigma^{xv}(k;\sigma)$. (A7c) again shows that $\tilde{k}^2$ is a root of

$$\tilde{k}^2(1 + a_v(\tilde{k}^2))^2 + M_e^2(1 + b_v(\tilde{k}^2))^2 = 0. \quad \text{(A8)}$$

Thus, it is a constant independent of $\mathbf{k}^2$. Besides, (A7a) implies that we have

$$\Sigma^{xv}_{HF}(\mathbf{k}, E_k;\sigma) = iM_e[a_v(\tilde{k}^2) - b_v(\tilde{k}^2)] \times(1 + a_v(\tilde{k}^2))^{-1}. \quad \text{(A9)}$$

Indeed, if we substitute (A7b) into (61), we find

$$(\gamma \cdot \mathbf{k} + i\gamma_4 E_k)u(ks) = iM_e \frac{1 + b_v(\tilde{k}^2)}{1 + a_v(\tilde{k}^2)} u(ks), \qquad \text{(A10)}$$

which gives (A9) immediately.

References

1. Walecka, J. D.: Theoretical Nuclear and Subnuclear Physics. Oxford: Oxford University Press 1995.
   Serot, B. D.: Rep. Prog. Phys. 55, 1855 (1992)
2. Bielajew, A. F., Serot, B. D.: Ann. Phys. 156, 215 (1984)
3. Brown, W. D., Puff, R. D., Wilets, L.: Phys. Rev. C2, 331 (1970)
4. Bielajew, A. F.: Nucl. Phys. A 404, 428 (1983)
5. Lee, T. D., Wick, G. C.: Phys. Rev. D9, 2291 (1974); Lee, T. D., Margulies, M.: Phys. Rev. D11, 1591 (1975)
6. Boguta, J., Bodmer, A. R.: Nucl. Phys. A292, 413 (1977)
7. Chin, S. A.: Ann. Phys. 108, 301 (1977)
8. Nakano, M., Hasegawa, A., Kouno, H., Koida, K.: Phys. Rev. C49, 3061 (1994). We are very grateful to G. Liu for calling our attention to this reference
9. Wu. S. S., Yao, Y. J.: Chin. Phys. Lett. 13, 261 (1996)
10. Lanske, H., Fuchs, C.: Phys. Lett. B345, 355 (1995); Ma, Z. Y., Chen, B. Q., Shi, H. L.: INPC'95. Singapore: World Scientific 1996
11. Bjorken, J. D., Drell, S. D.: Relativistic Quantum Fields. New York: McGraw-Hill 1965
12. Horowitz, C. J., Serot, B. D.: Nucl. Phys. A399, 529 (1983)
13. Serot, B. D., Walecka, J. D.: Adv. Nucl. Phys.16, 1 (1986)
14. Wu, S. S.: J. Phys. G16, 1447 (1990)
15. Kirson, M. W.: Ann. Phys. 66, 624 (1971); Wu, S. S.: Scientia Sinica 16, 347 (1973); *ibid.*17,468 (1974); Dickhoff, W. H., Faessler, A., Muether, H., Wu, S. S.: Nucl. Phys. A405, 534 (1983)
16. Jones, R. W., Mohling, F., Becker, R. L.: Nucl. Phys. A220, 45 (1974)
17. Lowdin, P. O.: Phys. Rev. 97, 1474 (1955)
18. Morse, P. M., Feshbach, H.: Methods of Theoretical Physics, Part I, p.459. New York: McGraw-Hill 1953